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Constrained Slews for Single-Axis Pointing

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Introduction

THERE is a significant interest in the determination of maneuver strategies for underactuated satellites, where the availability of only two torque components prevents the spacecraft from performing arbitrary slews. The problem is representative of those situations in which an actuator failure [1,2] or inherent physical constraints (e.g., use of magnetic torquers only as attitude effectors that deliver a command torque along directions perpendicular to the external magnetic field [3,4]) result in constraints onto the admissible rotations and full three-axis control is not attainable. In these cases, the available torque is constrained on a plane, perpendicular to the direction $\hat{\mathbf{b}}$ about which it is not possible to rotate the spacecraft, where $\hat{\mathbf{b}}$ is prescribed in either the body-fixed frame (for actuator failure) or in the fixed target frame (as for the direction of the magnetic field).

In the present note it is shown that, regardless of the direction of $\hat{\mathbf{b}}$, there always exists an eigenaxis rotation [5] that provides for the exact pointing of a given body-fixed axis toward a prescribed target direction, identified by the unit vectors $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\tau}}$, respectively. The direction of the admissible rotation eigenaxis and the amplitude of the resulting rotation are analytically determined for any possible combination of the unit vectors $\hat{\boldsymbol{\sigma}}$, $\hat{\boldsymbol{\tau}}$, and $\hat{\mathbf{b}}$.

In [6], a similar problem is dealt with, and its solution proposed as a means for planning feasible maneuvers for underactuated spacecraft, where the overall angular displacement ε from a specified target frame is minimized by rotating the body frame about a nonnominal Euler axis $\hat{\mathbf{g}}$. The analytical solution of Giulietti and Tortora [6] provides a reorientation strategy in which the final misalignment error grows with the angle between $\hat{\mathbf{e}}$ and $\hat{\mathbf{g}}$, where $\hat{\mathbf{e}}$ is the nominal eigenaxis. Thus, the optimal nonnominal eigenaxis $\hat{\mathbf{g}}$ lies along the projection of $\hat{\mathbf{e}}$ onto the plane of the admissible rotation axes, perpendicular to the direction of $\hat{\mathbf{b}}$, where the angle between $\hat{\mathbf{e}}$ and the admissible $\hat{\mathbf{g}}$ is minimum [6]. The resulting final pointing error grows linearly with the desired nominal rotation ϕ about $\hat{\mathbf{e}}$, when ϕ is small, reaching a limit value $\varepsilon = \cos^{-1}(\hat{\mathbf{g}} \cdot \hat{\mathbf{e}})$ for a desired rotation $\phi = \pi$. As a consequence, the misalignment error may attain rather large values when the angle between $\hat{\mathbf{g}}$ and $\hat{\mathbf{e}}$ and the required reorientation angle ϕ are both large.

The actual limits for the maximum tolerable attitude error will strongly depend on accuracy requirements for the considered application, but the exact acquisition of a prescribed attitude may not be mandatory, especially in the presence of failures, when a deterioration of system performance is expected and may be tolerated. Conversely, accurate pointing of a single direction $\hat{\boldsymbol{\sigma}}$, such as the boresight of a sensor payload or a directional antenna, may be vital for the mission or, at least, sufficient for a minimal level of operations in those critical situations where arbitrary attitude maneuvers cannot be accomplished. As an example, orbit maneuvers require pointing of the thruster nozzle with an accuracy within a fraction of a degree from the desired Δv direction, whereas the roll angle of the spacecraft around the thrust vector line has no effect on the outcome of the thrust pulse.

By application of an approach similar to that discussed in [6], based on the identification of a suitable eigenaxis $\hat{\mathbf{g}}$ and corresponding rotation $\hat{\phi}$, this paper demonstrates that it is possible to analytically define a rotation which exactly aims a body-fixed axis toward a prescribed (yet arbitrary) target direction. The approach is initially derived by choosing an ad hoc body reference frame $\mathcal{F}_B$, where the first axis of $\mathcal{F}_B$ is parallel to the unit vector $\hat{\boldsymbol{\sigma}}$, which must be pointed along the target direction $\hat{\boldsymbol{\tau}}$. This choice makes the derivation of the pointing strategy simpler from the mathematical standpoint. It will then be generalized for arbitrary sensor and target directions by use of a suitable coordinate transformation matrix.

The derivation of the eigenaxis rotation that allows for the desired alignment will be discussed in the next section. Some examples of the resulting rotation strategies will then be given in the Results section, where a comparison with the minimum misalignment provided by the solution proposed in [6] is also presented. The Conclusions section ends the note.

Maneuver Planning

As stated in the Introduction, the desired maneuver is represented by the rotation of a body-fixed axis $\hat{\boldsymbol{\sigma}}$ (representing the boresight of a sensor, an antenna, or a thruster nozzle) toward a prescribed target direction $\hat{\boldsymbol{\tau}}$. The minimum amplitude rotation that allows for the desired maneuver can be determined by means of an inverse cosine function, $\alpha = \cos^{-1}(\hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\tau}})$ with $0 \le \alpha \le \pi$. If $\alpha < \pi$, the rotation takes place in the plane Π that contains both $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\tau}}$, that is, around the unit vector $\hat{\mathbf{e}}_m = (\hat{\boldsymbol{\sigma}} \times \hat{\boldsymbol{\tau}})/(\|\hat{\boldsymbol{\sigma}} \times \hat{\boldsymbol{\tau}}\|)$. This situation is depicted in Fig. 1. When $\alpha = \pi$, the plane Π is not defined and the cross product $\hat{\boldsymbol{\sigma}} \times \hat{\boldsymbol{\tau}}$ is the null vector. In this case, the desired reorientation can be performed by means of a rotation of 180 deg about any axis perpendicular to the direction of $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\tau}}$. This latter, singular case will be considered at the end of this section. Note that, in this framework, the minimum rotation angle α is strictly positive. A reversal of the axis $\hat{\mathbf{e}}_m$, rather than a change in the sign of α, would result from a rotation of $\hat{\boldsymbol{\sigma}}$ in the opposite direction.

Given the preceding definitions, the target direction $\hat{\boldsymbol{\tau}}$ can be expressed as

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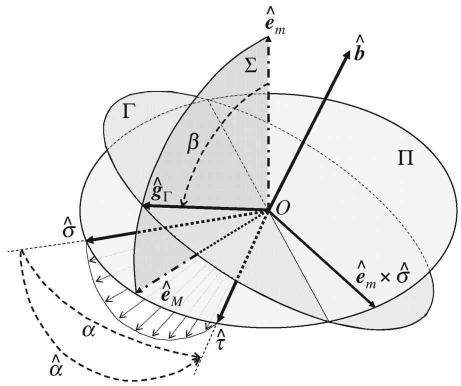


Fig. 1 Geometry of the problem.

$$\hat{\boldsymbol{\tau}} = \hat{\boldsymbol{\sigma}} \cos\alpha + (\hat{\mathbf{e}}_m \times \hat{\boldsymbol{\sigma}}) \sin\alpha \tag{1}$$

By indicating with $\hat{\mathbf{e}}_M$ the unit vector of the bisector of the angle α,

$$\hat{\mathbf{e}}_M = \frac{\hat{\boldsymbol{\sigma}} + \hat{\boldsymbol{\tau}}}{\|\hat{\boldsymbol{\sigma}} + \hat{\boldsymbol{\tau}}\|} = \frac{(1+\cos\alpha)\hat{\boldsymbol{\sigma}} + \sin\alpha(\hat{\mathbf{e}}_m \times \hat{\boldsymbol{\sigma}})}{\sqrt{2(1+\cos\alpha)}} \tag{2}$$

the desired rotation of $\hat{\boldsymbol{\sigma}}$ onto $\hat{\boldsymbol{\tau}}$ can be accomplished by rotating the body-fixed frame $\mathcal{F}_B$ about any axis $\hat{\mathbf{g}}$ on the plane Σ, perpendicular to Π, that contains both $\hat{\mathbf{e}}_m$ and $\hat{\mathbf{e}}_M$, that is, any unit vector $\hat{\mathbf{g}}$ lying along a linear combination of $\hat{\mathbf{e}}_m$ and $\hat{\mathbf{e}}_M$. This point is proven by deriving the expression of the angle of rotation $\hat{\alpha}$ for any possible vector $\hat{\mathbf{g}}$ on the plane Σ.

Letting β be the angle between $\hat{\mathbf{g}}$ and $\hat{\mathbf{e}}_m$ (where it is possible to bound the values of β to $-\pi/2 \le \beta \le \pi/2$), it is $\hat{\mathbf{g}} = \hat{\mathbf{e}}_m \cos\beta + \hat{\mathbf{e}}_M \sin\beta$, that is,

$$\hat{\mathbf{g}} = \left[\frac{\sin\beta}{\sqrt{2}}\sqrt{1+\cos\alpha}\right]\hat{\boldsymbol{\sigma}} + \left[\frac{\sin\beta}{\sqrt{2}}\sqrt{1-\cos\alpha}\right](\hat{\mathbf{e}}_m \times \hat{\boldsymbol{\sigma}}) + [\cos\beta]\hat{\mathbf{e}}_m \tag{3}$$

Noting that the three unit vectors $\hat{\boldsymbol{\sigma}}$, $(\hat{\mathbf{e}}_m \times \hat{\boldsymbol{\sigma}})$, and $\hat{\mathbf{e}}_m$ form a right-handed triad, it is possible, without loss of generality, to make them represent the initial position of the body axes, $\mathcal{F}_B^{(1)} \equiv (O; \hat{\mathbf{i}}_1, \hat{\mathbf{j}}_1, \hat{\mathbf{k}}_1)$. The body-frame components $(g_1, g_2, g_3)^T$ of the eigenaxis $\hat{\mathbf{g}}$ can thus be obtained from the coefficients between square brackets in Eq. (3).

The coordinate transformation matrix $\mathbb{T}_{21}$ from $\mathcal{F}_B^{(1)}$ to $\mathcal{F}_B^{(2)} \equiv (O; \hat{\mathbf{i}}_2, \hat{\mathbf{j}}_2, \hat{\mathbf{k}}_2)$ can be expressed in terms of the Euler eigenaxis rotation $\hat{\alpha}$ about the generic unit vector $\hat{\mathbf{g}} \in \Sigma$ as [5]

$$\mathbb{T}_{21} = \begin{bmatrix} c\hat{\alpha} + g_1^2(1-c\hat{\alpha}) & g_1g_2(1-c\hat{\alpha}) + g_3 s\hat{\alpha} & g_1g_3(1-c\hat{\alpha}) - g_2 s\hat{\alpha} \\ g_1g_2(1-c\hat{\alpha}) - g_3 s\hat{\alpha} & c\hat{\alpha} + g_2^2(1-c\hat{\alpha}) & g_2g_3(1-c\hat{\alpha}) + g_1 s\hat{\alpha} \\ g_1g_3(1-c\hat{\alpha}) + g_2 s\hat{\alpha} & g_2g_3(1-c\hat{\alpha}) - g_1 s\hat{\alpha} & c\hat{\alpha} + g_3^2(1-c\hat{\alpha}) \end{bmatrix} \tag{4}$$

where $c\hat{\alpha} = \cos\hat{\alpha}$ and $s\hat{\alpha} = \sin\hat{\alpha}$. Given the choice of axes as discussed, the final position of $\hat{\mathbf{i}}_1 \equiv \hat{\boldsymbol{\sigma}}$ is $\hat{\mathbf{i}}_2 \equiv \hat{\boldsymbol{\tau}}$, represented in $\mathcal{F}_B^{(2)}$ by the vector $\hat{\boldsymbol{\tau}}_2 = (1,0,0)^T$. From Eq. (1), the same unit vector is expressed as $\hat{\boldsymbol{\tau}}_1 = (\cos\alpha, \sin\alpha, 0)^T$ in $\mathcal{F}_B^{(1)}$ only if the eigenaxis rotation about $\hat{\mathbf{g}}$ performs the required rotation of $\hat{\boldsymbol{\sigma}}$. Thus, the following relation between the coordinates of $\hat{\boldsymbol{\tau}}$ must hold:

$$\hat{\boldsymbol{\tau}}_2 = \mathbb{T}_{21}\hat{\boldsymbol{\tau}}_1 \longrightarrow \hat{\boldsymbol{\tau}}_1 = \mathbb{T}_{12}\hat{\boldsymbol{\tau}}_2 = [\mathbb{T}_{21}]^T\hat{\boldsymbol{\tau}}_1$$

that is,

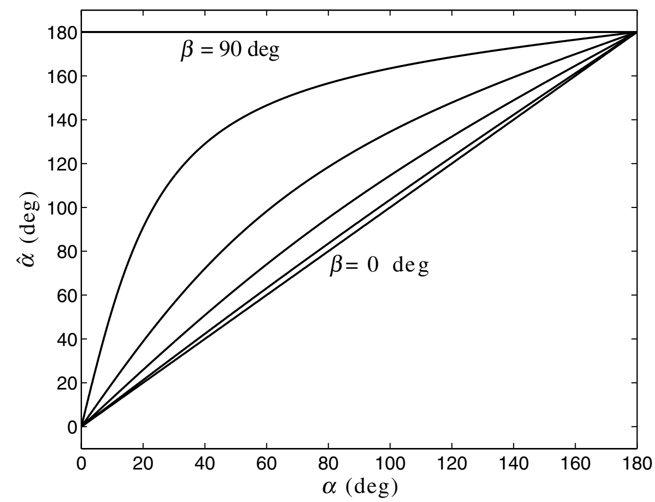


Fig. 2 Rotation angle $\hat{\alpha}$ as a function of α and β.

$$(\cos\alpha, \sin\alpha, 0)^T = (T_{11}, T_{12}, T_{13})^T$$

where T_{ij} is the (i,j)th component of the coordinate transformation matrix $\mathbb{T}_{21}$. From the equation $T_{13} = 0$, one gets

$$\sin\hat{\alpha} = \frac{g_1 g_3}{g_2}(1-\cos\hat{\alpha})$$

which can be substituted into the equation $T_{12} = \sin\alpha$, getting

$$(1-\cos\hat{\alpha}) = \frac{g_2}{g_1(g_2^2 + g_3^2)}\sin\alpha$$

Upon substitution of the expressions of the components of the eigenaxis $\hat{\mathbf{g}}$ from Eq. (3), and after some manipulation, one gets

$$\cos\hat{\alpha} = 1 - \frac{2\sin^2\alpha}{\sin^2\alpha\sin^2\beta + 2(1+\cos\alpha)\cos^2\beta} \tag{5}$$

which allows one to evaluate $\hat{\alpha}$ by means of an inverse cosine function. Figure 2 provides the plot of the function $\hat{\alpha} = \hat{\alpha}(\alpha, \beta)$ for $\beta = 0$, 20, 40, 60, 80, and 90 deg.

Note that, if $\beta = 0$ (that is, $\hat{\mathbf{g}} \equiv \hat{\mathbf{e}}_m$), Eq. (5) reduces to

$$\cos\hat{\alpha} = 1 - \frac{\sin^2\alpha}{(1+\cos\alpha)} = \cos\alpha$$

that holds for $\hat{\alpha} = \alpha$, as expected. Conversely, if $\beta = \pm\pi/2$ and $\hat{\mathbf{g}} \equiv \pm\hat{\mathbf{e}}_M$, the same equation reduces to $\cos\hat{\alpha} = -1$, that holds for $\hat{\alpha} = \pm\pi$. This means that, in this latter case, a maximum rotation of 180 deg is necessary in either direction to accomplish the desired rotation of the unit vector $\hat{\boldsymbol{\sigma}}$, during which the unit vector $\hat{\boldsymbol{\sigma}}$ itself draws half of a cone around $\hat{\mathbf{g}}$. As a final observation, one should also note that a rotation of $\hat{\alpha}^\star = -(2\pi - \hat{\alpha})$ in the opposite direction allows for the same rotation of $\hat{\boldsymbol{\sigma}}$ following a longer angular path of little practical interest.

The possibility of choosing $\hat{\mathbf{g}}$ arbitrarily in a plane to accomplish the required alignment of the unit vectors $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\tau}}$ is particularly interesting when the spacecraft cannot be rotated about an arbitrary eigenaxis (as for an underactuated satellite), that is, the admissible eigenaxis is constrained over a plane Γ, perpendicular to a given torqueless direction $\hat{\mathbf{b}}$.

For any constraint direction $\hat{\mathbf{b}}$, it is possible to choose a rotation axis $\hat{\mathbf{g}}_\Gamma$ that lies at the intersection of the planes Γ and Σ. This axis must satisfy the constraint equation

$$\hat{\boldsymbol{b}} \cdot \hat{\boldsymbol{g}}_\Gamma = 0 \tag{6}$$

Assuming that $\hat{\mathbf{b}} = b_1\hat{\boldsymbol{\sigma}} + b_2(\hat{\mathbf{e}}_m \times \hat{\boldsymbol{\sigma}}) + b_3\hat{\mathbf{e}}_m$, the component representation of $\hat{\mathbf{b}}$ in $\mathcal{F}_B^{(1)}$ is $\hat{\mathbf{b}}_1 = (b_1, b_2, b_3)^T$ and the constraint (6) is expressed by the condition

$$(1/\sqrt{2})[b_1\sqrt{1+\cos\alpha} + b_2\sqrt{1-\cos\alpha}]\sin\beta + b_3\cos\beta = 0 \tag{7}$$

that can be solved for $\tan\beta$,

$$\tan\beta = -\frac{b_3\sqrt{2}}{b_1\sqrt{1+\cos\alpha} + b_2\sqrt{1-\cos\alpha}} \tag{8}$$

The inclination of the admissible eigenaxis with respect to $\hat{\mathbf{e}}_m$ is thus easily identified as a function of the nominal (minimum amplitude) rotation angle α about $\hat{\mathbf{e}}_m$ and the direction of the torqueless direction $\hat{\mathbf{b}}$. Note that the use of a four-quadrant inverse tangent is not necessary, as it is possible to bound the value of β between $-\pi/2$ and $\pi/2$, the same direction being achieved by either adding or subtracting π to β. In this way, the direction of the shortest rotation $\hat{\alpha}$ always matches that of the nominal rotation α about $\hat{\mathbf{e}}_m$, being $\hat{\mathbf{e}}_m \cdot \hat{\mathbf{g}}_\Gamma \geq 0$.

The direction of the torqueless axis $\hat{\mathbf{b}}$ is identified by means of the angles λ and θ, where θ represents the angle between $\hat{\mathbf{b}}$ and the plane Π, and λ is the angle between the projection of $\hat{\mathbf{b}}$ on Π and the initial position of the axis $\hat{\boldsymbol{\sigma}}$. The components of $\hat{\mathbf{b}}$ in the selected body frame $\mathcal{F}_B^{(1)}$ are thus given by $\hat{\mathbf{b}}_1 = (\cos\lambda\cos\theta, \sin\lambda\cos\theta, \sin\theta)^T$. Noting that, by changing the sign of the components of $\hat{\mathbf{b}}$, the same plane Γ of admissible rotation axes is considered, it is possible to span only half of the unit sphere by limiting the variation of θ between 0 and $\pi/2$ rad.

By substituting Eq. (8) into Eq. (3), and after some simple trigonometric manipulation, the eigenaxis for the required rotation, compatible with the constraint posed by the torqueless direction, is expressed as $\hat{\mathbf{g}}_\Gamma = \mathbf{g}_\Gamma/\|\mathbf{g}_\Gamma\|$, with

$$\begin{aligned}\mathbf{g}_\Gamma = {} & -b_3(1+\cos\alpha)\hat{\boldsymbol{\sigma}} - b_3\sin\alpha(\hat{\mathbf{e}}_m \times \hat{\boldsymbol{\sigma}}) \\ & + [b_1(1+\cos\alpha) + b_2\sin\alpha]\hat{\mathbf{e}}_m\end{aligned} \tag{9}$$

As discussed earlier, when $\alpha = \pm\pi$ the Γ and Π planes are not defined, but any axis perpendicular to $\hat{\boldsymbol{\sigma}}$ allows for a 180 deg rotation that takes $\hat{\boldsymbol{\sigma}}$ onto $\hat{\boldsymbol{\tau}}$. For this singular case, it is sufficient to choose $\mathbf{g}_\Gamma = \pm(\hat{\boldsymbol{\sigma}} \times \hat{\mathbf{b}})$, with $\hat{\alpha} = \alpha = \pm\pi$.

The prescribed alignment of $\hat{\boldsymbol{\sigma}}$ with $\hat{\boldsymbol{\tau}}$ can always be accomplished exactly, even the in presence of a direction about which it is not possible to rotate the spacecraft. The fact that the eigenaxis $\hat{\mathbf{g}}_\Gamma$ is invariant during the rotation, in both the moving and the fixed frames, guarantees that a control torque is always available about $\hat{\mathbf{g}}_\Gamma$, regardless of the fact that no torque is available about either a body-fixed or inertially fixed direction $\hat{\mathbf{b}}$, inasmuch as the mutual position of $\hat{\mathbf{g}}_\Gamma$ and $\hat{\mathbf{b}}$ is not affected by a rotation about $\hat{\mathbf{g}}_\Gamma$ itself. The worst case scenario is $\hat{\mathbf{b}} \equiv \mathbf{e}_m$, which requires a rotation of 180 deg about $\hat{\mathbf{g}}_\Gamma = \hat{\mathbf{e}}_M$.

The aforementioned procedure can be generalized to any body-axis choice. Letting $\hat{\boldsymbol{\sigma}}_B = (\sigma_1, \sigma_2, \sigma_3)^T$ and $\hat{\mathbf{e}}_{m_B} = (e_1, e_2, e_3)^T$ be the components of $\hat{\boldsymbol{\sigma}}$ and $\hat{\mathbf{e}}_m$, respectively, expressed in a generic body frame $\mathcal{F}_B = (O; \hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$, it is possible to project the components of all the considered unit vectors $\hat{\boldsymbol{\tau}}$, $\hat{\mathbf{b}}$, and $\hat{\mathbf{g}}_\Gamma$ in the ad hoc frame $\mathcal{F}_B^{(1)}$ by means of the coordinate transformation matrix:

$$\mathbb{T}_{1B} = [\mathbb{T}_{B1}]^T = [\hat{\boldsymbol{\sigma}}_B \vdots \hat{\mathbf{e}}_{m_B} \times \hat{\boldsymbol{\sigma}}_B \vdots \hat{\mathbf{e}}_{m_B}]^T \tag{10}$$

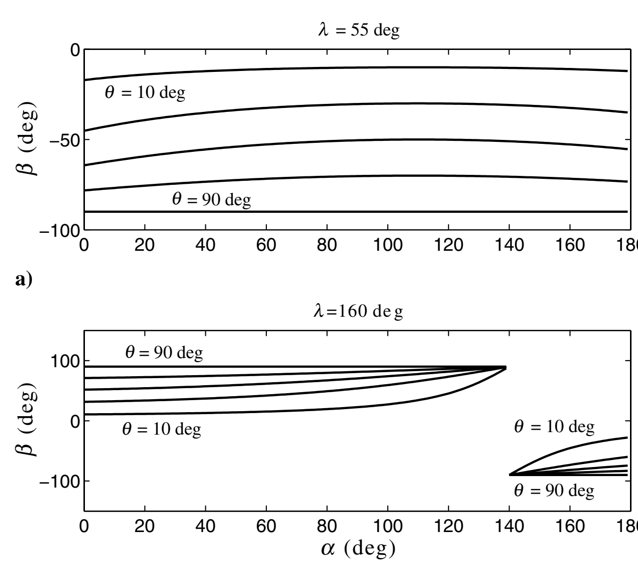
It is then possible to determine the admissible eigenaxis $\hat{\mathbf{g}}_\Gamma$ in $\mathcal{F}_B^{(1)}$ by means of the procedure outlined earlier, and obtain its body-frame components in $\mathcal{F}_B$ by means of the coordinate transformation matrix $\mathbb{T}_{B1}$.

Results

To investigate the features of the proposed reorientation planning technique in the presence of constraints on admissible rotations, the variation of β with α for different values of θ (10, 30, 50, 70, and 90 deg) is reported in Figs. 3a and 3b, for $\lambda = 55$ deg and $\lambda = 160$ deg, respectively. The first case (Fig. 3a) is representative of those situations when $0 < \lambda < 90$ deg or $-180 < \lambda < -90$, in which cases $|\beta|$ grows monotonically with θ, being $\beta < 0$ in the first case and $\beta > 0$ in the second one.

When $90 < \lambda < 180$ deg or $-90 < \lambda < 0$ deg, the sign of β depends on whether the bisector $\hat{\mathbf{e}}_M$ of the angle α lies between $\hat{\boldsymbol{\sigma}}$ and the intersection of the planes Π and Γ or not. This situation is reported in Fig. 3b ($\lambda = 160$ deg), where $\beta > 0$ if $\alpha/2 < \lambda - 90$ deg, whereas it is negative otherwise. For $\alpha/2 = \lambda - 90$ deg, the intersection of the planes Π and Γ lies along the bisector $\hat{\mathbf{e}}_M$ and a singular case is encountered, where the only admissible rotation axis that allows for the prescribed rotation of $\hat{\boldsymbol{\sigma}}$ is $\hat{\mathbf{e}}_M$ itself, with $\beta = \pm 90$ deg and $\hat{\alpha} = \pm 180$ deg, independent of the value of θ. A similar situation holds for $-90 < \lambda < 0$, where β is negative for $\alpha/2 < \lambda + 90$ deg, and positive otherwise, with $\beta = \pm 90$ deg and $\hat{\alpha} = \pm 180$ for $\alpha/2 = \lambda + 90$ deg. Note that, for $\theta = 90$ deg, it is $\hat{\mathbf{b}} \equiv \hat{\mathbf{e}}_m$ and $\beta = \pm 90$ deg, with $\hat{\alpha} = 180$ deg for all the required rotation angles α (Fig. 2).

A comparison with the technique discussed in [6] is considered, where the nominal eigenaxis rotation $(\hat{\mathbf{e}}, \phi)$ is assumed to be equal to the minimum amplitude rotation that takes $\hat{\boldsymbol{\sigma}}$ onto $\hat{\boldsymbol{\tau}}$ $(\hat{\mathbf{e}}_m, \alpha)$. In this case, the final desired attitude is given by the target frame $\mathcal{F}_T = (O; \hat{\boldsymbol{\tau}}, \hat{\mathbf{e}}_m \times \hat{\boldsymbol{\tau}}, \hat{\mathbf{e}}_m)$. Figure 4 reports the sensor axis pointing error and the overall misalignment error ε for the two techniques as a function of the nominal rotation α, for $\lambda = 45$ deg and $\theta = 50$ deg. It is possible to note how the total misalignment error achieves larger values, if pointing of a single direction is pursued, but the final pointing error of the sensor axis $\hat{\boldsymbol{\sigma}}$ is always zero, as expected. Conversely, the pointing error of the axis $\hat{\boldsymbol{\sigma}}$ is on the same order of magnitude of the overall misalignment error, which grows linearly for a small desired angular displacement and achieves values as high



a) b)

Fig. 3 Angle β (inclination of $\hat{\mathbf{g}}_\Gamma$ with regard to $\hat{\mathbf{e}}_m$) as a function of $\hat{\alpha}$ and $\hat{\mathbf{b}}$.

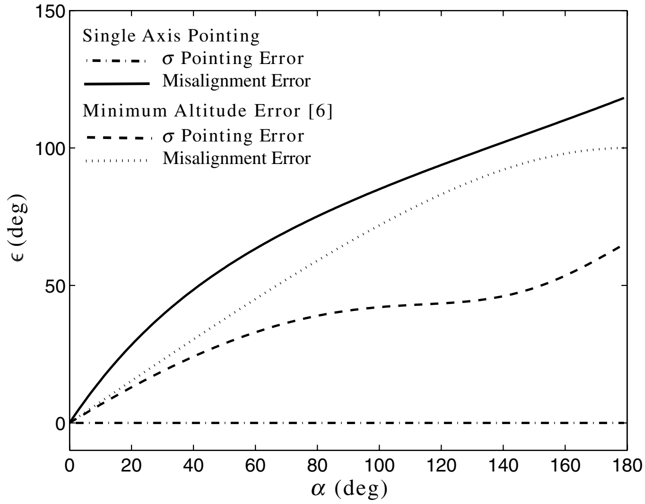


Fig. 4 Comparison of the errors for single-axis pointing strategy ($\hat{\alpha}, \hat{g}_\Gamma$) and minimum attitude error [6] ($\hat{\phi}, \hat{g}$).

as 65 deg for $\alpha = 180$ deg, when $\varepsilon = 100$ deg is also large and unacceptable for any application of practical interest.

Conclusions

A simple geometric framework for attitude maneuver planning of underactuated spacecraft was derived. A rotation eigenaxis allowing exact pointing of a body-fixed axis toward an arbitrary direction in space is determined in the presence of a constraint on admissible rotations. The resulting maneuver is identified by means of two angles β and $\hat{\alpha}$, representing the position of the admissible rotation eigenaxis and amplitude of the rotation, respectively. The proposed approach can be adopted for planning reorientation maneuvers in those cases when failures and/or physical constraints of the control system prevent the acquisition of an arbitrary desired attitude, but the exact pointing of a prescribed direction is mandatory for the mission.

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